

Flow Separation from the Smooth-Contoured Streamlined Surfaces

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Abstract

In this article the flow features of liquids and gases in the diffuser channels are analyzed on the basis of Prandtl-averaged equation for the boundary layer. It is shown that in this situation there is a layer of liquid or gas close to the wall where the shear stress τ increases in the direction along the normal to streamlined surface ($\partial\tau / \partial y > 0$), which enables the separation-free flow of working fluids in

diffuser channels. Such a flow becomes impossible downstream of the section where the shear stress τ and its derivative $\partial\tau / \partial y$ falls to zero.

On this basis, an effective way of preventing the flow separation from the streamlined walls in the wide angle diffusers is suggested.

Keywords: diffuser, longitudinal pressure gradient, shear stress, velocity, finned surface, boundary layer

1 Introduction

The problem of flow separation from the smooth-contoured streamlined surfaces is a major problem among the challenges of fluid and gas dynamics, in both theory and practice [4, 6, 19, 22], as when the separation occurs in a critical way the flow pattern is totally changed [5, 7, 11].

The flow separation occurs and it is an unavoidable consequence when steady flow turns into unsteady [1, 17]. If motion of working medium takes place inside the channels, during the flow separation from its walls the hydraulic resistance increases dramatically [16, 20], non-recoverable energy losses grow up, acoustic radiation increases and dynamic forces on all surfaces located in the area of moving medium rise sharply [2, 10, 18].

Due to the mentioned negative effects associated with the separation of working fluid mediums from the walls of different shape channels the problem of unseparated flow preservation in the flow sections of different devices and equipments becomes of utmost importance [3, 15].

However, for resolving at least a part of this problem it is necessary to have a clear idea of the mechanism of flow separation from the streamlined surfaces.

Unfortunately, in most publications the occurrence of flow separation is considered on a qualitative level with involvement of simple mechanical analogies.

In view of primary importance of the problem of flow separation from the smooth-contoured streamlined surfaces it makes sense of bringing literally those modest explanations of physics factors of the flow separation occurrence which are contained in well know publications.

2 A critical review of the traditional interpretations of the causes of flow separation from the smooth-contoured streamlined surfaces

Despite the importance of the problems outlined above, just a few sentences is given in all publications, starting from Prandtl down to our days, about the mechanism of occurrence of the flow separation from smooth surfaces.

Thus, L. Prandtl [12] points out that “the backward motion of the boundary layer can occur only with a sufficient degree of flow deceleration. This is because the wall, along which a boundary layer is formed, decelerates the fluid particles contacted with it, and an external flow, on the contrary, drags the fluid particles of the boundary layer contacted with it. Therefore, if the velocity of the streamlined

wall flow decreases very gradually, then increase of the boundary layer particles by an external flow is still sufficient to prevent the occurrence of reverse flow. Such a drag of boundary layer by the external flow is getting stronger when the motion in the boundary layer is turbulent rather than laminar”.

This intuitive interpretation of the mechanism of liquids and gas motion in the diffuser regions leaves out the interaction picture of external flow relative to boundary layer with boundary layer itself. As on the outside of boundary layer the shear stress is close to zero, in both laminar and turbulent boundary layers, then also no drag action of external flow should be in this area.

G. S. Samoylovich notes [13]: “separation of the boundary flow of viscous fluid can occur where there is a positive pressure gradient ($dp/dx > 0$). However, when the positive pressure gradient is small enough, the boundary layer separation may not occur if its kinetic energy store will be enough to overcome the diffuser region. It should be noted that the separation not only depends on the total amount of kinetic energy in the layer, but also on the friction law as the particles near the wall have a very low kinetic energy and may overcome the positive pressure gradient due to the shear forces between them and the outside part of the boundary layer”.

It is obvious that in this context the Prandtl’s formulation is extended on the inner part of the boundary layer where the shear stresses is acting and, accordingly, where is the interaction between adjoining layers of flowing medium.

L.G. Loitsiansky, referring to the mechanism of flow separation from the streamlined surface, in [9] points out that “in diffuser region the fluid moves from the region of lower pressure to the region of higher pressure against the differential pressure decelerating it. If fluid would be perfect and the velocity on the surface would not be equal to zero, then the kinetic energy store of the fluid would have been sufficient to overcome the decelerating field pressures. The pressure field in boundary layer does not differ from the pressure field in perfect fluid, meanwhile in close proximity to the surface the velocities are low, and the kinetic energy of fluid particles is negligible. Under these conditions, deceleration stops the fluid flow and reverses its flow direction under the action of the differential pressure directed against the flow. When the forward flow meets the backward fluid flow moving in the boundary layer, it results in sudden separation of the fluid flow lines from the solid body, thickening of the boundary layer and forcing it off from the surface”.

In the present interpretation, the flow separation from the streamlined surface is associated only with the amount of kinetic energy in the near-wall region, which should provide a fluid flow against the pressure growing in the backward direction.

It is easy to show that, in principle, there can be impossible to ensure the unseparated fluid flow in the diffuser regions if we associate the possibility of unseparated flow only with the kinetic energy store in the various sections of the boundary layer. Indeed, if the longitudinal pressure gradient $\partial p / \partial x$ is determined from the motion equation for a perfect fluid (for one-dimensional flow $\frac{dp}{dx} = -\rho u_t \frac{du_t}{dx}$, where u_t – longitudinal velocity on the outer edge of the boundary layer) and in a fixed section the pressure across the boundary layer does not change ($dp/dx = 0$), then at any distance $y < \delta$ (δ – the boundary layer thickness) under any flow condition in the boundary layer the unseparated flow will be impossible as in

region of $y < \delta$ the kinetic energy of flow will be always less than on the outer edge of the boundary layer.

Also, the statement that the backward flow occurs after the separation point under the influence of differential pressure directed against the flow does not correspond to the physical picture of separation.

In actuality, after the separation point the longitudinal positive pressure difference becomes close to zero and after the separation point fluid moves streamwise without the pressure increase.

In practical terms, the same reason explanation of the flow separation from the wall is contained in the book of N. F. Krasnov, V. N. Koshevoy and V. T. Kalugin [8]. “The flow in boundary layer has a lower velocity as compared with the main flow stream and, therefore, it is exposed to influence of relatively large negative acceleration. The mechanical energy of the fluid particles near the wall is small and their ability to move in the direction of increasing pressure is limited. There comes a moment when the energy store may be insufficient to overcome the positive pressure gradient because some energy continuously transits into heat due to the shear forces. Therefore, at first, the fluid flow near the surface slows down, and then changes its direction. The separation of the fluid flow line and, as a consequence, the separation of the boundary layer from the surface occurs as a result of the reverse flow formation”.

Note, that here as in other listed sources, the flow separation is interpreted as the result of the main flow interaction with the reverse flow along surface influenced by the continuing positive pressure action.

As already mentioned above, after the separation point $dp / dx \approx 0$. This conclusion follows directly from the equations of motion. It can also be shown from these equations that when $y = 0$, on the wall always is $\frac{dp}{dx} = \frac{d\tau}{dy}\bigg|_{y=0}$.

In other words, when the main flow comes closer to the separation point, $\frac{d\tau}{dy}\bigg|_{y=0}$ automatically goes to zero and dp / dx also comes to zero with the same extent.

The reverse flows, after the separation point, occur due to the ejecting property of the separated flow.

Interpretation of the flow separation given by H. Schlichting in [14] does not differ by its originality. “If there is a region of increasing pressure along the body contour, then, in general, the fluid slowing down in the boundary layer and, therefore, having a small kinetic energy is unable to move too far into the region of high pressure. Instead, it deflects away from the high pressure region, detaches when this happens from the body and pushes from the wall into the external flow. Aside from that, near the wall the stopped fluid particles induced by the pressure gradient usually move in the reverse direction of the external flow. We will determine the separation point as the point between the forward and backward flow in the layer close to the wall”.

Comparing all above information about the causes of flow separation from the smooth-contoured streamlined surfaces we can note almost complete identity of all given above reasons (except for the wording of Prandtl), the essence of which can be expressed in one sentence only.

The flow separation from the smooth-contoured streamlined surfaces occurs in the diffuser flow region and in that section where the kinetic energy of fluid moving in the boundary layer is not enough to overcome the pressure difference acting in the reverse direction of the fluid flow.

3 Basic physics of the mechanism of flow separation from the smooth-contoured surfaces

Information importance of the given interpretation of reasons of the boundary layer separation from the smooth-contoured surface is limited by two reasonably evident conditions in which the boundary layer separation occurs in the diffuser region under positive pressure gradient ($dp / dx > 0$), and the higher the kinetic energy of the flow near the streamlined surface, the higher the value of longitudinal pressure gradient dp / dx when the flow separation occurs.

The last conclusion explains the later separation of the turbulent boundary layer comparing with the laminar layer where the kinetic energy of the fluid particles is substantially less than in the turbulent velocity profile.

However, the existing presentation of the reasons of boundary layer separation from the streamlined surfaces are not based on the analysis of changes of all actual stress factors that occur when the working fluids move in the diffuser regions, and do not explain the reason of unseparated flow near the wall where the pressure difference continuous acting in the reverse direction is higher than the kinetic energy moving in the near-wall region of the fluid.

In this regard, we will consider the problem of flow separation on the basis of analysis of the stress factors determining the fluid flow in boundary layer.

The classical interpretation of the mechanism of boundary layer separation from the streamlined surfaces in the diffuser flow region, given above, is actually based on an inertial motion analogy of the solid bodies moving upward.

In this case, same as in the diffuser fluid flow, the inertial forces, the shear forces and the gravity force component directed in the reverse direction of the solid body motion are acting on the moving body. However, when it moves in any fixed position the velocity of all points of the body remains constant, and the shear forces act only along the plane of contact of the body surface with the conjugated surfaces.

The situation is fundamentally different in moving fluids and gases. In cross-section of the boundary layer the velocities, and shear stresses as well, are changing with distance from the streamlined surface and the nature of these changes is determined by the sign of the acting longitudinal pressure gradient.

Accordingly, the analysis of force factors determining the flow of working medium within the boundary layer should be conducted on the basis of Prandtl-averaged equations for the two-dimensional flow [21]:

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{dp}{dx} + \frac{\partial \tau}{\partial y}, \quad \frac{\partial p}{\partial y} = 0, \quad (1)$$

where u and v – projection of a velocity vector \vec{c} onto coordinate axes;

$\tau = \tau_m + \tau_t$ – total shear stress within the boundary layer;

$\tau_m = \mu \frac{\partial u}{\partial y}$ – molecular shear stress;

μ – viscosity coefficient;

τ_t – and the turbulent shear stress.

As defined, on the outer edge of the boundary layer $\partial \tau / \partial y = 0$ and equation (1) goes over into Euler equations for the two-dimensional flow of the perfect fluid:

$$\left. \begin{aligned} \rho \left(u_t \frac{\partial u_t}{\partial x} + v_t \frac{\partial u_t}{\partial y} \right) &= - \frac{dp}{dx} \\ \rho \left(u_t \frac{\partial v_t}{\partial x} + v_t \frac{\partial v_t}{\partial y} \right) &= - \frac{dp}{dy} \\ \frac{\partial u}{\partial x} + v_t \frac{\partial v}{\partial y} &= 0 \end{aligned} \right\} \quad (2)$$

Solving the equation system (2) for the particular channel allows finding the pressure gradient distribution in this channel and, in particular, the distribution of the longitudinal pressure gradient $\partial p / \partial x$ along its walls.

Since, in the cross-section of the boundary layer, according to the second Prandtl's equation, the pressure does not change ($\partial p / \partial y = 0$), then distribution of pressure and its longitudinal gradient determined in the specified manner can be taken as the first approximation for the total selected cross section of this layer.

Further, one can see from the equation (1) that on the streamlined surface:

$$\frac{dp}{dx} = \frac{\partial \tau}{\partial y} \Big|_{y=0} \quad (3)$$

In other words, the sign of longitudinal pressure gradient $\partial p / \partial x$ is the same as the sign of transverse gradient of the shear stress on the wall.

This condition is very important in the analysis of the changes that occur within the boundary layer when coming closer to the section where the flow separation from the streamlined wall occurs.

At multiplication of equation (1) by the fluid element $dV = dx dy dz$ we obtain the balance equation of all forces acting on the elementary fluid particles moving within the boundary layer:

$$\rho dx dy dz \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{dp}{dx} dx dy dz + \frac{\partial \tau}{\partial y} dx dy dz \quad (4)$$

We will present this equation as:

$$\frac{dp}{dx} dx dy dz = \frac{\partial \tau}{\partial y} dx dy dz - \rho dx dy dz \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) \quad (5)$$

Here, the external force $dF_p = \frac{dp}{dx} dx dy dz$ due to the longitudinal pressure gradient is balanced by shear forces $dF_\tau = \frac{\partial \tau}{\partial y} dx dy dz$ and inertial forces $dF_u = \rho dx dy dz \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right)$.

In the given equation (5) the elementary external force

$$dF_p = dF_\tau - dF_u \quad (6)$$

If the force F_p , according to the second Prandtl's equation, does not change in the cross section of the boundary layer, then the values dF_τ and dF_u are always changing in such a way that their algebraic sum had remained constant in any cross section of the boundary layer and equal to the external force dF_p .

Accordingly, the fluid motion along the streamlined surface is unseparated until the specified condition is fulfilled, i.e., until the diagram deformation of the force distribution dF_τ and dF_u under the influence of external (in this case geometric) forces provides compensation of the force dF_p .

The qualitative change of the distribution diagram of the shear stresses in the cross section of the boundary layer can be obtained on the basis of three boundary conditions mentioned below:

- 1) For $y = 0$, $\partial \tau_w / \partial y = dp / dx$
- 2) For $y = \delta$, $\tau = 0$
- 3) For $y = \delta$, $\partial \tau / \partial y = 0$

The qualitative dependency diagrams $\tau = f(y)$, plotted taking into account these conditions, for convergent ($dp / dx < 0$), gradientless ($dp / dx = 0$) and diffuser ($dp / dx > 0$) flows are given in Figure 1.

Comparing these diagrams of the shear stresses we can note the fundamental difference between these stress distribution in convergent and diffuser flows.

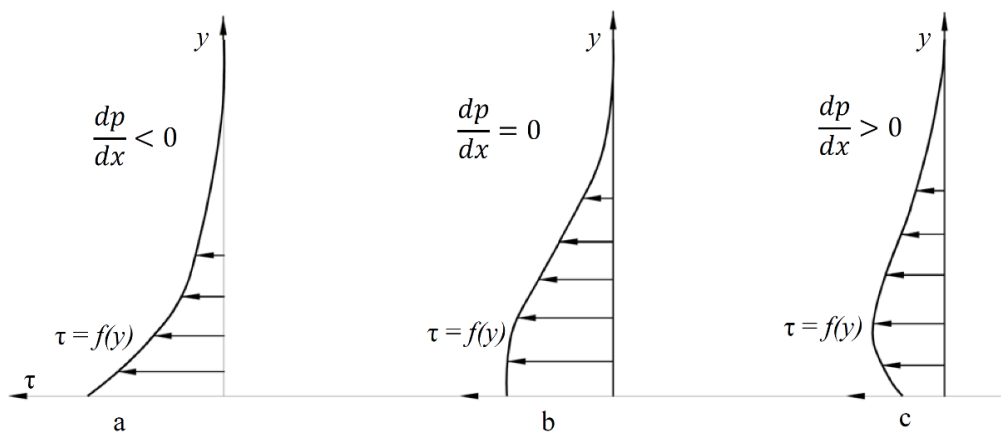


Fig. 1: Diagrams of the shear stress distribution for:
a – convergent, b – gradientless, c – diffuser flows within the boundary layer

If the shear stresses in confusor are maximum directly on the wall and farther they are monotonically decreasing to zero at the external edge of the boundary layer, then the maximum stresses in diffusers are located at some distance y_c from the streamlined surface and with distance from the wall these stresses are increasing at $y < y_c$, and they start decreasing only at $y \geq y_c$. Accordingly, the transverse stress gradient in confusors $\partial\tau / \partial y < 0$ and with distance from the wall it monotonically decreases to zero.

In diffusers at $y < y_c$ the value $\partial\tau / \partial y$ is increasing with distance from the wall ($\partial\tau / \partial y > 0$), and then at $y > y_c$ it starts monotonically decreasing ($\partial\tau / \partial y < 0$).

The corresponding dependence diagrams $\partial\tau / \partial y = f(y)$ for three considered types of flow are given in Figure 2.

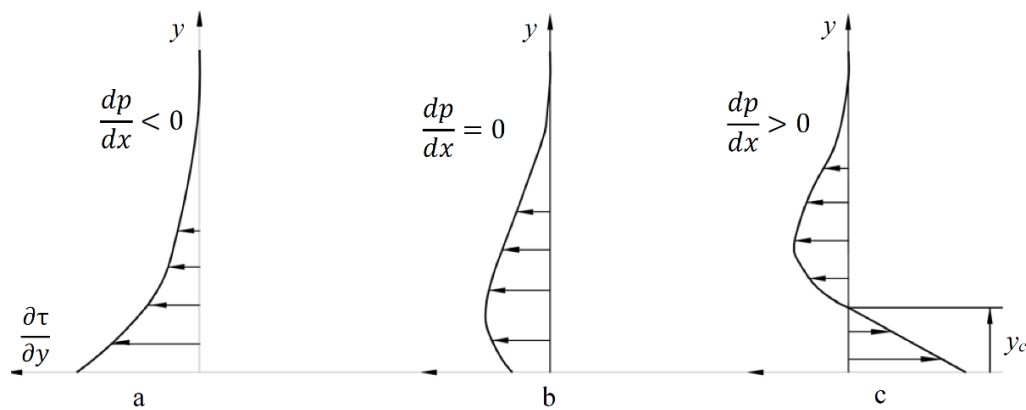


Fig. 2: Diagrams of the transverse shear stress distribution for:
a – convergent, b – gradientless, c – diffuser flow in the boundary layer

Thus, the shear forces dF_τ in cross section of the boundary layer are qualitatively changing also. Conclusion that the shear forces in the diffuser flow region at distance $y < y_c$ are directed in the same direction as the fluid flow is an essential result of the above analysis.

In other words, at this distance the overlying layers drag the underlying (stopped) fluid layers, hence, providing the principal possibility of unseparated flow at a positive pressure gradient (see Prandtl's definition of separation).

If we add up the distribution of all forces acting within the boundary layer, then for the unseparated flow the balance of stress factors shown in Figure 3 should always be realized.

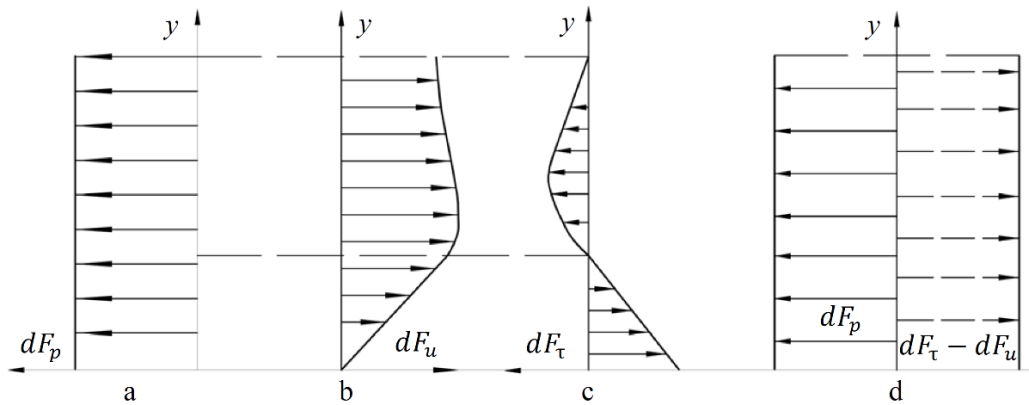


Fig. 3: Diagrams of the stress factors acting in the boundary layer with diffuser flow:

a – change of a longitudinal pressure gradient in relation to the transverse coordinate y , b – change of a transverse shear stress gradient in cross section of the boundary layer, c – change of the inertial forces in cross section of the boundary layer, d – balance diagram of acting forces in cross section of the boundary layer.

Distance from the wall y_c at which a positive value of the transverse shear stress gradient (shear force dF_τ) that makes possible flowing of the fluid in the near-wall region against the force dF_p acting in the opposite direction remains the same depends directly on a value of the positive pressure gradient dp/dx . The greater the longitudinal pressure gradient dp/dx , the greater the value of the coordinate y_c . In other words, value y_c determines the response level of the moving fluid medium to the external (in this case geometric) influence determined by the pressure gradient dp/dx .

However, the possibility of a flow response to external influence has certain limits which, if exceeded, cause a flow separation from the streamlined surface.

In addition, the known conditions in the separating point of the boundary layer (shear stress on the wall ($\tau_w = 0$) and the transverse velocity gradient ($du/dy = 0$) in separation point should be equal to zero), are to be supplemented with the condition of equality to zero on the wall of the transverse shear gradient ($\left.\frac{\partial \tau}{\partial y}\right|_{y=0} = 0$). If the first two conditions can be referred to the conditions necessary for the flow separation, then the value $\left.\frac{\partial \tau}{\partial y}\right|_{y=0}$ that equals to zero is already a sufficient condition for the flow separation.

It is worthwhile to say that the possibility of unseparated flow in the diffuser region at shear stress on the streamlined surface equals to zero first time was shown in [9] in 1968, where this situation is considered to be a condition for getting the highest energy recovery factor in diffusers.

The same can be concluded theoretically from the given analysis of all force factors acting within the boundary layer. To ensure the unseparated flow the transverse shear stress gradient $\partial \tau_w / \partial y$ is important, and not an absolute value of the shear stress, as specifically a value of the transverse shear stress gradient deter-

mines the shear force on the streamlined wall and a value of the shear force determines possibility of the fluid motion against the longitudinal pressure gradient acting in diffusers.

The transition from unseparated flow to separated flow occurs in a critical way as a result of impossibility to fulfil the main boundary condition in some section of channel $dp/dx = \frac{\partial \tau}{\partial y} \Big|_{y=0}$.

As already mentioned above, with increase of the longitudinal pressure gradient dp/dx the distance from the wall y_c increases, and the sign of derivative $\partial \tau / \partial y$ changes. At the same time, the maximum value $\frac{\partial \tau}{\partial y} \Big|_{y=y_c}$ increases also. At some value dp/dx a further increase of coordinate y_c and maximum value of shear stress becomes impossible, and the regime in critical way changes from unseparated regime to regime with separation of the boundary layer from the streamlined surface.

Thus, the flow separation from the streamlined surface is its crisis condition in which all possible structural changes in the boundary layer providing possibility of the fluid flow against a positive pressure gradient defined by the geometrical shape of the channel or the geometrical shape of the streamlined body are completely exhausted.

Analyzed physical phenomenon of the flow separation from the streamlined surface and the mathematical formulation of a condition of its unavoidable occurrence $(\frac{\partial \tau}{\partial y} \Big|_{y=0})$ will allow not only to provide more reasonable physical interpretation of the flow separation prevention in a well-known way but also to develop the new effective methods of influence on the flow regime in the different type diffusers.

4 Conclusions

1. This review of literature shows that mostly the same qualitative model of separation is used for the description of the mechanism of flow separation from the smooth-contoured streamlined surfaces, based on the analogy of the solid body motion to the top of the inclined plane.

2. On the basis of the averaged Prandtl's equations for the fluid flow within the boundary layer the balance equation for all elementary forces acting on the fluid particles in the boundary layer was composed, and it was shown that in the diffuser flow regions (at positive pressure gradients) a region with the positive value of the transverse shear stress gradient $(\partial \tau / \partial y > 0)$ is formed directly near the wall, which provides possibility, due to the drag motion of viscous forces, for the underlying fluid layers to flow against the pressure difference acting in the opposite direction in that region where the kinetic energy of the fluid is close to zero.

3. This study provided the basis for next formulation of the flow separation from the streamlined surface.

Flow separation from the smooth-contoured streamlined surfaces is its critical

condition in which the possibilities of moving medium to change the diagrams of inertial shear forces providing a flow response to an external geometric influence determined by the longitudinal positive pressure gradient are totally exhausted.

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